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HYDRODYNAMICS AND HEAT EXCHANGE IN VISCOUS INCOMPRESSIBLE LIQUID FLOW BETWEEN DISKS ROTATING IN A CYLINDRICAL CASING

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The article provides the results of a numerical investigation of convective heat exchange in a closed system consisting of two coaxial disks rotating at the same angular velocity within an immobile cylindrical casing.

Improvement in the reliability of calculations of the thermal and thermal-stress conditions of turbomachine rotors requires, in particular, greater accuracy in assigning the boundary conditions of heat exchange at their end-face surfaces. Most of the theoretical, including numerical, investigations in this field were based on mathematical models utilizing cut-off parabolic Navier—Stokes equations. However, in many actual channels adjacent to the rotor surface, inertial forces produce return flow, which cannot be calculated by using methods based on boundary layer theory. A simplified model of one of such systems would be a cavity formed by an immobile cylindrical casing and two disks rotating at a constant angular velocity. This problem has been solved in [1-3] for small Reynolds numbers ($Re < 2 \cdot 10^3$) in the absence of heat exchange.

In stating our problem here, we use the same simplifying assumptions as in [2, 3]: Steady-state laminar flow is contemplated, the velocity and temperature fields are assumed to be axisymmetric, and the thermophysical characteristics of the medium are considered to be constant.

The flow geometry and the coordinate system are shown in Fig. 1.

The system of differential equations of convective heat exchange, written in terms of dimensionless variables, can be conveniently reduced to four equations with identical structures [4]:

$$a\left\{\frac{\partial}{R\partial R}\left(b\frac{\partial}{\partial R}(c\Phi)\right)+\frac{\partial^2\Phi}{\partial Z^2}\right\}-\frac{1}{R}\left\{\frac{\partial}{\partial R}(RV_r\Phi)+\frac{\partial}{\partial Z}(RV_z\Phi)\right\}+d=0, \tag{1}$$

where ω/R, RV_φ, Ψ, and T are considered as the sought function Φ. The corresponding values of the coefficients a, b, c, and d determining the actual form of the equations in the system are given in Table 1.

The r_0 and $2\pi n r_0$ values are used as the coordinate and velocity scales, respectively.

The solution of the system of equations (1) must satisfy the following boundary conditions:

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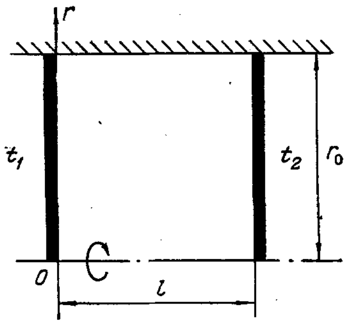

Fig. 1. Flow geometry and the coordinate system.

TABLE 1. Coefficients of Equation (1)

Φ	a	b	c	d
$\frac{\omega}{R}$	$\frac{1}{\mathrm{Re}}$	$\frac{1}{R}$	R^2	$\frac{1}{R^2}\frac{\partial V_\varphi^2}{\partial Z}$
RV_φ	$\frac{1}{\mathrm{Re}}$	R^3	$\frac{1}{R^2}$	0
Ψ	1	R^3	$\frac{1}{R^2}$	$R\omega$
T	$\frac{1}{\mathrm{RePr}}$	R	1	0

$$\Psi = 0,\quad RV_\varphi = R^2,\ T = 1,\qquad Z = 0,$$
$$\Psi = RV_\varphi = 0,\quad T = 1 - Zr_0/l,\qquad R = 1,$$
$$\Psi = 0,\quad RV_\varphi = R^2,\quad T = 0,\qquad Z = l/r_0,$$
$$\Psi = RV_\varphi = \partial T/\partial R = 0,\qquad R = 0.$$

The boundary conditions for vorticity at the disk and casing surfaces and at the symmetry axis are determined according to the method proposed in [5].

In solving numerically system (1), the Seidel method is used in combination with the lower relaxation after the equations have been integrated with respect to an elementary cell of the difference grid [4].

The relaxation coefficients are determined from the stability conditions and are corrected on the basis of numerical experiments. The Poisson equation for the stream function is solved by determining the relaxation coefficients in accordance with [6].

The convective terms are approximated by using a "hybrid" scheme consisting of a combination of central and directional differences [7].

One of the causes of divergence of the iteration process in calculations, which has been noted in [2], is the violation of one of the integral relationships of continuous media kinematics — the Stokes equation. This results in the appearance of additional sources devoid of physical meaning, which have a purely difference origin; they increase considerably near the symmetry axis of the flow. The difficulties connected with the violation of the conservative nature of the difference scheme in a cylindrical coordinate system were resolved in [8]. The relationships derived there were used for the difference approximation of the initial equations (1).

We tested the described method earlier during a numerical investigation of viscous liquid flow with heat exchange in a rotating radial convergent channel [9].

The above calculation method was used to solve our problem for larger values of the Reynolds number than in previous investigations. The Re = 10^5 value was used as an upper limit on the basis of the fact that, for Reynolds numbers of this order, transition from laminar flow conditions to turbulent flow is observed in systems similar to the described one

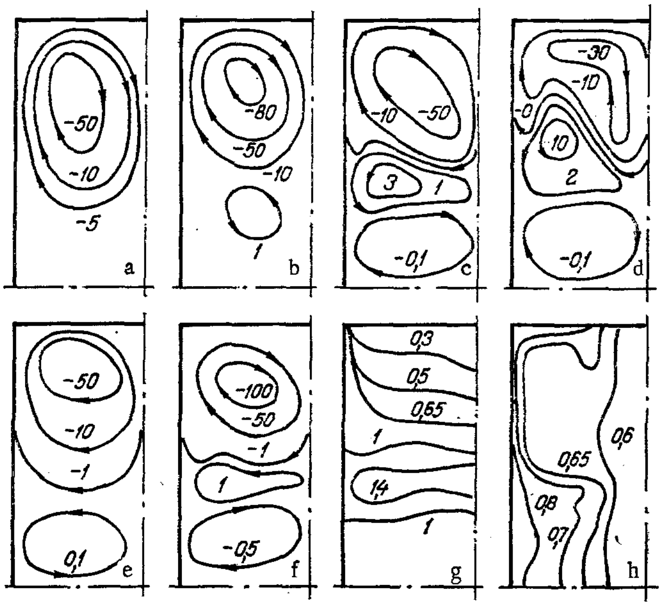

Fig. 2. Distribution of dependent variables in the analyzed region: stream isolines $\Psi \cdot 10^4$; L = 1; a) Re = 10^2; b) 10^3; c) 10^4; d) 10^5; e) L = 0.5; Re = 10^4; f) L = 0.75; Re = 10^4; g) isolines of the velocity vector's peripheral component RV_φ/R^2 (L = 1; Re = 10^5); h) isotherms (L = 1; Re = 10^5).

[10]. Since, for small L values, the liquid between the disks rotates virtually as a solid [3], we investigate here the $0.5 \leqslant L \leqslant 1$ range, which is characterized by a substantial recirculation flow.

Only uniform grids were used in calculations, since, in contrast to external flow around bodies, it is difficult to separate a priori regions of grid node bunching for the flow under consideration. A series of calculations for L = 1 on a 31 × 31 grid was performed first. Passage to a new, 41 × 41, grid did not produce a substantial change in the general character of flow in the cavity between the disks, except for insignificant variations in the integral characteristics, and this grid was, therefore, adopted as the basic one.

Since the stability of the computational process depends considerably on the initial approximation, the fields of dependent variables obtained earlier for smaller Re numbers and stored in the external computer memory were used for each of the variants.

The iteration process was discontinued when the following condition was satisfied:

$$\max_{ij} \left| \frac{\Phi_{ij}^{p+1} - \Phi_{ij}^{p}}{\Phi_{ij}^{p}} \right| < 10^{-3},$$

where p is the iteration number. The vanishing of the total thermal flux through the surfaces bounding the analyzed region was used as an additional convergence criterion. The existence of flow symmetry relative to the vertical plane passing through the middle of the cavity served as the physical accuracy criterion for the solutions.

The variation in the convective flow structure as the Reynolds number increases is shown in Fig. 2 for L = 1. The fields of dependent variables are shown with an allowance for symmetry only for the left-hand half of the region under analysis. We can conventionally separate two different forms of recirculation flow. For Re < $3 \cdot 10^2$, the flow is characterized by a two-vortex structure. Subsequently, two additional toroidal vortices arise near the symmetry axis, the intensity and the meridional cross-sectional area of which increase with the Reynolds number, whereas the intensity and the meridional cross-sectional area of the external vortices diminish, while they themselves are forced back toward the peripheral boundary of the cavity. For Re $\geqslant 5 \cdot 10^3$, a flow region with very weak recirculation develops near the rotation axis, where the liquid rotates virtually as a solid. If the disk rotation frequency increases still further, this region expands and, for Re = 10^5, encompasses the lower third of the cavity.

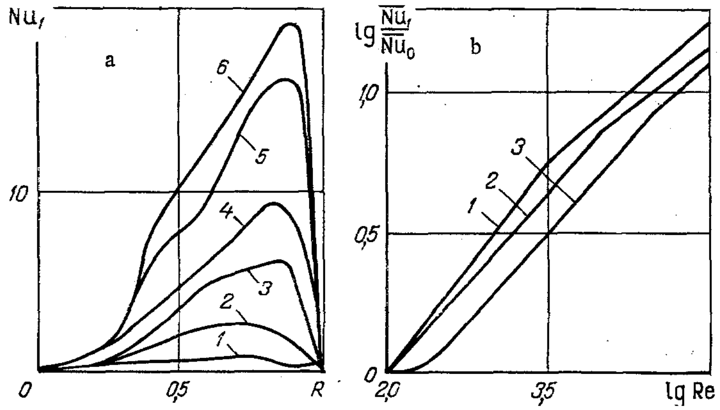

Fig. 3. Local (a) and mean (b) heat transfer at the heated disk surface. For a: 1) Re = 100; 2) 10^3; 3) $5 \cdot 10^3$; 4) 10^4; 5) $5 \cdot 10^4$; 6) 10^5; for b: 1) L = 1; 2) 0.75; 3) 0.5.

The streamline distribution calculated for Re = 1000 is in good qualitative and quantitative agreement with those obtained earlier in [2, 3] with uniform 11 × 11 and 16 × 16 grids. The fields of isolines of the velocity vector's peripheral component and of the isotherms complete our general idea of the character of the flow. It is evident that the region of elevated local heat transfer at the disk surface and the region of the highest vortex intensity in the cavity are mutually related (Fig. 3a).

Calculations have shown that similar variation of the convective flow structure in the cavity between the disks generally occurs also for other relative width values in the investigated range. At the same time, the limiting Reynolds number Re* for which a four-vortex structure develops increases as L decreases. The intensity of additional vortices and their meridional cross-sectional area decrease (Fig. 2e and f), so that this structure apparently does not arise for $L < 0.5$.

One should expect on the basis of physical considerations that the intensity of the basic recirculation flow is at a maximum for a certain relative width of the clearance between the disks L_m. Actually, as was mentioned above, the spinning of the liquid by the disks for small values of L conforms to the rotation law for a solid, and secondary flow is absent. On the other hand, the surface area of the casing — the braking factor in this hydrodynamic system — increases for large values of L, which also reduces the secondary flow intensity. Analysis of the calculation results suggests that the value of L_m is close to 0.75.

The dimensionless drag torque coefficient of the disks is determined by the expression [3]

$$c_m = -\frac{4\pi}{\mathrm{Re}} \int_0^1 \frac{\partial (V_\varphi R)}{\partial Z} R^3 dR.$$

The calculation results are satisfactorily approximated by the following relationship:

$$c_m = -16.87 L^{-2\lg L - 0.471} \mathrm{Re}^{-0.861}.$$

The temperature field established in the process of calculations is used to determine the dimensionless thermal fluxes at the disk and casing surfaces, which are characterized by mean Nusselt numbers

$$\overline{\mathrm{Nu}}_1 = -2 \int_0^1 \left(\frac{\partial T}{\partial Z}\right)_{Z=0} R dR, \quad \overline{\mathrm{Nu}}_2 = 2 \int_0^1 \left(\frac{\partial T}{\partial Z}\right)_{Z=L} R dR,$$

$$\overline{\mathrm{Nu}}_3 = 2 \int_0^1 \left(\frac{\partial T}{\partial R}\right)_{R=1} dZ, \quad \overline{\mathrm{Nu}}_0 = 1/L.$$

Under steady-state conditions, the thermal balance relationship $\sum_{i=1}^{3}\overline{\mathrm{Nu}}_i = 0$ holds: it was satisfied with an accuracy to 0.01 for each of the calculated temperature fields. For the chosen thermal boundary conditions, the relationships $\overline{\mathrm{Nu}}_1 = \overline{\mathrm{Nu}}_2$ and $\overline{\mathrm{Nu}}_3 = 0$ were satisfied for all the calculated variants.

Figure 3b shows the variation of the mean heat transfer from a heated disk with an increasing Reynolds number for different relative widths of the clearance between the disks. Each of the curves has a pronounced inflection for Re numbers close to Re*. The development of an additional vortex pair near the horizontal symmetry axis and its effect on the heat transfer due to the initial recirculation flow (as a result, the temperature of the liquid particles washing the peripheral regions rises) reduce the rate of increase in the mean heat transfer from the disk surface. It should be noted that a similar effect was observed in experimental investigations of thermal convection in a rotating closed cylindrical cavity for the same thermal boundary conditions involving a multivortex structure of secondary flow [11].

The calculation results are satisfactorily approximated by the following system of similarity equations:

$$\mathrm{Re} < 10^2 \quad \overline{\mathrm{Nu}}_1/\overline{\mathrm{Nu}}_0 = 1,$$

$$10^2 \leqslant \mathrm{Re} \leqslant 10^4 \quad \overline{\mathrm{Nu}}_1/\overline{\mathrm{Nu}}_0 = 0.16\,\mathrm{Re}^{0.43+0.18\lg L},$$

$$10^4 \leqslant \mathrm{Re} \leqslant 10^5 \quad \overline{\mathrm{Nu}}_1/\overline{\mathrm{Nu}}_0 = 0.00873 L^{-1.73\lg L}\mathrm{Re}^{1.096-0.089\lg\mathrm{Re}}.$$

The accuracy of the relationships describing convective heat exchange in the investigated system is limited essentially by the scope of application of the chosen mathematical model of liquid flow, which, in turn, can probably be determined only by physical experiments. As these are lacking, the results obtained for large Reynolds numbers ($\mathrm{Re} > 10^4$) would probably require refinement by taking into account the turbulent transfer effects and introducing additional measures for reducing the errors due to difference approximation.

NOTATION

r_0, outside radius of disks; l, casing width; n, rotation frequency; t_1 and t_2, temperatures of the disk surfaces ($t_1 > t_2$); r and z, radial and axial coordinates, respectively; V_z, V_r, and V_φ, axial, radial, and tangential components of the velocity vector, respectively; $Z = z/r_0$; $R = r/r_0$; $L = l/r_0$; $T = (t - t_2)/(t_1 - t_2)$, temperature; $\mathrm{Re} = 2\pi n r_0^2/\nu$, Reynolds number; ν, kinematic viscosity; $\mathrm{Pr} = \nu\rho c_p/\lambda$, Prandtl number; ρ, density; λ, thermal conductivity coefficient; c_p, specific heat at constant pressure; $\omega = \partial V_r/\partial Z - \partial V_z/\partial R$, tangential component of the velocity vortex; Ψ, stream function; $RV_r = -\partial\Psi/\partial Z$; $RV_z = \partial\Psi/\partial R$; c_m, dimensionless drag torque coefficient of the disks; $\mathrm{Nu} = \alpha r/\lambda$, local Nusselt number; $\overline{\mathrm{Nu}} = \bar{\alpha} r_0/\lambda$, mean Nusselt number; α and $\bar{\alpha}$, local and mean heat exchange coefficients, respectively. Subscripts: 1, left-hand disk; 2, right-hand disk; 3, casing; 0, case involving only thermal conductivity (Re = 0); m, maximum value.

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SOLUTION OF LIMITING PROBLEMS OF EXPANDING FLOWS OF GASEOUS SUSPENSIONS

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UDC 532.529.5

The equations of equilibrium expanding flows of a gaseous suspension with an arbitrary volume concentration of particles are transformed into the equations of an expanding ideal gas and, in particular, in the two-dimensional stationary case, into the linear Chaplygin equations.

Studies of limiting motions, for example motions with equal velocities of the soil and of the gas expanding into its pores, or equilibrium flows, when not only the velocities but also the temperatures are equal, help to elucidate the important qualitative features of the flows of dispersed media and sometimes permit obtaining results with satisfactory accuracy.

The motion of an equilibrium mixture is described, generally speaking, by the system of equations for a single-phase, continuous, imperfect-gas medium (see, for example, [1]). The perfect-gas equations are obtained from this system only in the case of a low volume concentration of particles and in the absence of phase transitions, which permits applying the analytical apparatus of classical gas dynamics [1, 2].

The results of this work follow from the representations of the mechanism of the phenomena given by S. A. Khristianovich in his research on the properties of dispersed flows for the example of nonstationary one-dimensional flows of soil and gas contained in its pores [3, 4].

We study below motion in a space with interphase heat exchange. For large volume concentrations of particles, we restrict ourselves to expansion flows $d\rho/dt$.

In gas dynamics, an example of an expanding flow is the motion of a gaseous suspension in a nozzle.

The equations of an ideal perfect pseudogas with an arbitrary volume concentration of particles, in particular, in the two-dimensional stationary case — the linear Chaplygin equations, are obtained for describing the limiting states of expanding flows of gaseous suspensions.

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